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TEACHING MATERIAL ON

MATHEMATICS

SCHOOL OF SCIENCE

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Defn. :- Dynamic Programming

Dynamic Programming is recent development in mathematical technique which is useful in optimizing a sequence of interrelated ~~complex~~ decisions by using recursion to solve a complex problems with ~~overall~~ effectiveness over a given period of time.

Differences between LPP and DPP following Problems may be solved:- DPP (Allocation, Inventory Control and replacement)

LPP	DPP
1) LP has its set of procedure (algorithm) to solve any decision-problems.	1) DPP has no any procedure (algorithm), as such, to solve a decision Problem. rather it break the given problem into a sequence of easier and smaller sub problems which are then solved in a sequential orders called stages.
ii) LP approach considers one time period (stage) solution to a given problem.	ii) Whereas DP approach is better known for getting its optimal solution for decision making over a pd. of time - slices.
(iii) In LPP there is a restriction that it can't handle many optimization problems which involves a large number of decision variable and/or a large number of inequality constraints.	(iii) Such problems are solved by the technique of Dynamic Programming in an orderly manner by starting from one stage to the next and is completed after the final stage is reached.

(2) Optimality in Dynamic Programming :—
(or Bellman's Principle of optimality in Dynamic programming) :—

Richard Bellman was the first to develop the technique of Dynamic Programming in early 1950, who gave the Principal of optimality as under :—

"An optimal policy (set of decisions) has the property that whatever be the initial state and initial decisions, the remaining decision must constitute an optimal policy for the state resulting from the first decision."

we note that a problem which does not satisfy the principle of optimality cannot be solved by dynamic programming.

(3) Multistage Decision Problems :—

Such a problem in which decisions have to be made at successive stage is called a multistage decision problems. which can be categorize on the basis of their properties as follows :—

1). The outcome of a dynamic programming problems may be deterministic or stochastic (probabilistic). If the stage of process is given, the outcome of the decision at any stage is uniquely determined then it is called as deterministic case and if there is a set of possible outcomes given by

a known probability distribution then (3)
it is called as stochastic case.

ii) The no. of possible decisions at any stage, from which we have to choose one may be finite or infinite.

iii) The total no. of stages in the process may be known or unknown according as according as they may be finite or infinite.

(4) Characteristic of a D.P.P (Dynamic Programming Problem)

The basic features of a DPP are :—

i) The problem can be fragmented/divided up into different stages, with a policy decision required at each stages.

ii) Each stage has a number of states assumed with it.

(iii) The effect of the policy decision at each stage is to convert the current state into a state associated with the next stage.

(iv) Given the current stage, an optimal policy for the remaining stages is independent of the policy adopted in previous stages.

(v) The solution procedure begins by finding the optimality condition (policy) for each of the state of the last stage.

(vi) A recursive relationship (functional equation) is provided which indentifies the optimal policy for each stage with r stages remaining, given the optimal policy for each stage with exactly (n-1) stages remaining.

(vii) The solution procedure moves backwards, stage by stage - each time finding the optimal policy for each ~~stag~~ state of that stage while using a recursive relationship (functional equation) until it finds the optimal policy when starting at the initial stage.

④ ⑤ Method of solution of a Multi-stage problem by Dynamic Programming with finite number of stages :—

Step-1:- We formulate the problem and then develop the functional equations :- (i.e, a recursive relation connecting the optimal decision function for the n stage problem with the optimal decision function for the (n-1) stage sub-problem n = 1, 2, 3, ... n is developed.)

Step-2) Next we solve the functional equations for determining the optimal policy. For this we write the optimal decision function for one stage sub problem and solve it. Then next we solve the optimal decision function for 2-stages, 3-stages, --- (n-1) stages & n-stage problem.

—: Illustrative Examples: —

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Example-1 A positive quantity c is to be divided into n parts in such a way that the product of the n parts is to be a maximum. Obtain the optimal subdivision.

Soln:- Let us write the classical formulation of the problem:- If the number c is divided into n-parts (say) $x_1, x_2, x_3, \cdots x_n$, then to find $x_1, x_2, x_3 \cdots x_n$ in such a way that it

$$\text{Maximize } z = x_1 x_2 x_3 \cdots x_n \text{ such that}$$
$$x_1 + x_2 + \cdots + x_n = c \quad \text{——— (1)}$$

Now to solve the problem by dynamic programming we proceed as follows:—

Step-1: To develop the functional equations ("Recursive Relation"):—

First of all we develop a recursive relation connecting the optimal decision function for the n-stage problem with the optimal decision function for the $(n-1)$ stage sub problems, $n = 1, 2, \cdots n$

Let x_i ($i = 1, 2, \cdots n$) be the ith part of c, Here each part i may be considered as a stage. Since x_i may assume any non-negative values such that $x_1 + x_2 + \cdots + x_n = c$. Hence, the alternatives at each stage are infinite. Thus it is the problem of a continuous system and so the optimal decision at each stage are determined by the use of old classical technique (ie, by differential calculus method).

Let $f_n(c)$ denote the maximum attainable product, when the quantity or number c is divided into n-parts. Therefore f_n depends on n because the quantity c is fixed. Hence, $f_n(c)$ is a function of the discrete variable n ($n = 1, 2, \cdots n$)

⇒ P.T.O. ⇒

Now if $n=1$, and if c is divided into one part only then we have $x_1 = c$ — (1) and hence, 3/10 $f_1(c) = c$ — (2)

Similarly if $n=2$ and if c is divided into two parts y_1 and y_2 then let us assume that $y_1 = z$ ∴ $y_2 = c-z$ and hence,

$$f_2(c) = \text{Max}\{y_1 \cdot y_2\} = \underset{0\le z\le c}{\text{Max}}\{z(c-z)\}$$

or

$$f_2(c) = \underset{0\le z\le c}{\text{Max}}\{z f_1(c-z)\} \quad \text{— (3)}$$

$$\{\text{since } f_1(c-z) = c-z \text{ from (2)}\}$$

Now for $n=3$ if c is divided into three parts say x_1, x_2, x_3 then $x_1 = z$ (say) ∴ $x_2 + x_3 = c-z$

∴ the part $c-z$ is further divided into x_2 and x_3 whose maximum attainable product is $f_2(c-z)$ by the definition of $f_n(c)$.

$$\therefore f_3(c) = \text{Max.}\{x_1 \cdot x_2 \cdot x_3\} = \underset{0\le z\le c}{\text{Max}}\{z f_1(c-z)\} \quad \text{— (4)}$$

Proceeding in this way we get the recursive relation (called functional equation) for $n=m$ (say) as under:—

$$f_m(c) = \underset{0\le z\le c}{\text{Max}}\{z f_{m-1}(c-z)\} \quad \text{— (5)}$$

$\boxed{\text{Step-2}}$: To solve the functional equation so obtained by using differentiation to determine optimal policy:—

From equation (2), we have $f_1(c) = c$

from eqn (3)

$$f_2(c) = \underset{0\le z\le c}{\text{Max}}\{z f_1(c-z)\}$$

$$= \underset{0\le z\le c}{\text{Max}}\{z(c-z)\} = \underset{0\le z\le c}{\text{Max}}(zc - z^2)$$

Now $z(c-z)$ is Max^m ~~or minimum~~ for that value of z for which $\frac{d}{dz}\{z(c-z)\} = c-2z$ (must vanish) $=0$

$$\therefore z = \frac{c}{2}, \quad c-z = c - \frac{c}{2} = \frac{c}{2}$$

$z(c-z)$ is max at $z = \frac{c}{2}$, since $\frac{d^2}{dz^2}(zc - z^2) = -2$ (Negative) — (6)

$\therefore f_2(c) = \frac{c}{2} \cdot \frac{c}{2} = \left(\frac{c}{2}\right)^2$ and the optimal policy for two parts is $\left(\frac{c}{2}, \frac{c}{2}\right)$, i.e., division of c into two equal pa

have

From equation (4) i.e, $f_3(c) = \max\limits_{0 \le z \le c} \{ z f_2(c-z) \}$ (1)

311 $= \max\limits_{0 \le z \le c} \left\{ z \left(\frac{c-z}{2} \right)^2 \right\}$

[$\because$ using eqn (6) i.e, $f_2(c) = \left(\frac{c}{2}\right)^2$

so $f_2(c-z) = \left(\frac{c-z}{2}\right)^2$.]

Now $z \cdot \left(\frac{c-z}{2}\right)^2$ is max.^{m} for that value of z, for which

$\frac{d}{dz}\left\{ z\left(\frac{c-z}{2}\right)^2 \right\} = 1 \cdot \left(\frac{c-z}{2}\right)^2 - z\left(\frac{c-z}{2}\right) = 0$ on solving we get $z = \frac{c}{3}$, $c - z = \frac{2c}{3}$

i.e, $c - z = \frac{2c}{3}$ is to be divided into two parts whose product is Maximum.

By the policy for two parts $f_2(c-z)$ i.e, $f_1\left(\frac{2c}{3}\right)$ is attained when the two parts are $\frac{1}{2}\left(\frac{2c}{3}\right)$ and $\frac{1}{2}\left(\frac{2c}{3}\right)$ i.e $\frac{c}{3}, \frac{c}{3}$.

$\therefore f_3(c) = \frac{c}{3}\left\{ \frac{c - (c/3)}{2} \right\}^2 = \left(\frac{c}{3}\right)^2$

and the optimal policy for three parts is $\left(\frac{c}{3}, \frac{c}{3}, \frac{c}{3}\right)$ i.e, division of c in three equal parts.

The computations at stages 1, 2, 3 shows that by analogy in general for n parts (stages), we have the optimal policy is $\left(\frac{c}{n}, \frac{c}{n}, \cdots, \frac{c}{n}\right)$

and hence $f_n(c) = \left(\frac{c}{n}\right)^n$ ——— (7)

This result can be proved by induction.

Basis step: Assuming that the result (7) holds for $n = 2, 3, \cdots, m$

for which $f_m(c) = \left(\frac{c}{m}\right)^m$

Induction step :- Now we shall prove that (7) also holds for $n = m+1$ Then from (5) we have,

$f_{m+1}(c) = \max\limits_{0 \le z \le c} \{ z f_m(c-z) \}$

$= \max\limits_{0 \le z \le c} \left\{ z \left(\frac{c-z}{m}\right)^m \right\}$

Next $z\left(\frac{c-z}{m}\right)^m$ is Max.^{m} for the value of z for which

$\frac{d}{dz}\left\{ z \cdot \left(\frac{c-z}{m}\right)^m \right\} = \underbrace{1 \cdot \left(\frac{c-z}{m}\right)^m - z\left(\frac{c-z}{m}\right)^{m-1} \cdot \frac{1}{m}}_{\text{solving we get}}$ (vanishes) i.e $= 0$

$\therefore \boxed{z = \frac{c}{m+1}}$

Thus we have $z = \frac{c}{m+1}$

$\therefore f_{m+1}(c) = \frac{c}{m+1}\left\{\frac{c - \frac{c}{m+1}}{m}\right\}^m = \left(\frac{c}{m+1}\right)^{m+1}$

and so the optimal policy in this case is

$\left(\frac{c}{m+1}, \frac{c}{m+1}, \ldots, \frac{c}{m+1}\right)$

Hence the required optimal policy for the index n is $\left(\frac{c}{n}, \frac{c}{n}, \ldots, \frac{c}{n}\right)$ Ans.

Example-2 Use dynamic-programming to solve that $-\sum_{i=1}^{n} p_i \log p_i$, subject to $\sum_{i=1}^{n} p_i = 1$ is maximum when $p_1 = p_2 = \cdots = p_n = \frac{1}{n}$.

Sol. Left for you people. (Try yourselves)

Thanks.

Example 3 Find Min $Z = x_1 + x_2 + \cdots + x_n$, when $x_1, x_2, \ldots x_n = d$, $x_1, x_2, \ldots, x_n \geq 0$

Example 4 Use the principle of optimality to compute the maximum value of $b_1x_1 + b_2x_2 + b_3x_3 + \cdots + b_nx_n$ when $x_1 + x_2 + \cdots + x_n = c$ and $x_i \geq 0$ when $i = 1, 2, \ldots n$.

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Example 5. By dynamic Programming technique, solve the problem Maximize $Z = x_1^2 + x_2^2 + x_3^2$ subject to the constraint $x_1 + x_2 + x_3 \geq 15$, $x_1, x_2, x_3 \geq 0$.

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Dynamic Programming Terminology

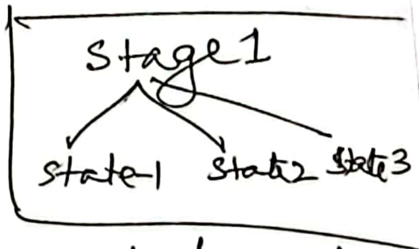
Some terms are common in every problems. They are :-

1. Stage :— Any problem of Dynamic programming can be decomposed or divided into a sequence of smaller subproblems which are called as "stage".

At each stage there are some number of decision alternatives or course of action which are appropriately selected. Stages generally represents different time intervals, places or other things depending upon the problem.

For example in the replacement problem year is a stage. Each territory represents in salesman problem as a stage etc.

2. State : Each Stage in a dynamic programming problem has a certain number of states associated with it.



The states represents a number of various conditions of the decision process. The ~~variables~~ or decision variables by which we represents the conditions or status of the system in any stage are called "State-variables".

Stages ~~~~ of the decision-making process there could be a finite or infinite number of ~~~~ "states". Ex If we consider the shortest route problem then a particular city is considered as state variable in any stage of this problem.

3. Return function :— A return function is an algebraic equation which describes either worth (or benefit) associated with every decision, which affect the state of the system at the next stage corresponding to each stage. and this decision help in arriving at the optimal solution at the current stage.

The return function depends on the state variables as well as the decision made at a particular stage "

4. Optimal Policy :— An optimal policy or a decision at a stage gives optimal (maximum or minimum) return for a given value of the state variable.

Some more Problems on Dynamic Programming

Q1. Determine the value of x_1, x_2, and x_3 so as to Maximize $Z = u_1 \cdot u_3 \cdot u_3$

Subject to the constraints

$$x_1 + x_2 + x_3 = 10 \text{ and } x_1, x_2, x_3 \geq 0.$$

Soln:- Let us define state variable

u_j ($J = 1, 2, 3$) such that

$u_3 = x_1 + x_2 + x_3 = 10$ at stage 3

$u_2 = u_3 - x_3 = x_1 + x_2$ at stage 2

$u_1 = u_2 - x_2 = x_1$ at Stage 1

The maximum value of Z for any feasible value of state variable is given by

$$f_3(u_3) = \underset{x_3}{\text{Max}} \{x_3 \cdot f_2(u_2)\}$$

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$$f_2(u_2) = \max_{x_2}\{x_2 \cdot f_1(u_1)\}$$

$$f_1(u_1) = x_1 = u_2 - x_2$$

Thus $f_2(u_2) = \max_{x_2}\{x_2 \cdot (u_2 - x_2)\}$

$$= \max_{u_2}\{x_2 u_2 - x_2^2\} \quad \text{— (1)}$$

Differentiating (1) w.r.t x_2 and equating to zero which will be the necessary condition for maximum or minimum value of a f^2), we have

$$u_2 - 2x_2 = 0 \text{ or } x_2 = u_2/2$$

Now using Bellman's principle of optimality, we get

$$f_2(u_2) = (u_2/2) \cdot u_2 - (u_2/2)^2 = u_2^2/4$$

and $f_1(u_3) = \max_{u_3}\{x_3 \cdot f_2(u_2)\}$

$$= \max_{u_3}\{x_3 (u_2^2/4)\}$$

$$= \max\left\{x_3 \cdot \frac{(u_3 - x_3)^2}{4}\right\}$$

Also, differentiating (1) or $f_2(u_2)$ w.r.t u_3 and equating to zero, we get,

$$\frac{1}{4}\left\{x_3 \cdot 2(u_3 - x_3)(-1) + (u_3 - x_3)^2\right\} = 0$$

$$(u_3 - x_3)(-2x_3 + u_3 - x_3) = 0$$

$$(u_3 - x_3)(u_3 - 3x_3) = 0$$

Now either $x_3 = u_3$ which is trivial as $x_1 + x_2 + x_3 = u_3$ or $u_3 - 3x_3 = 0$.

or $x_3 = u_3/3 = 10/3$.

$\therefore x_2 = \frac{u_2}{2} = \frac{u_3 - x_3}{2} = \frac{1}{2}\left(10 - \frac{10}{3}\right) = \frac{10}{3}$

$x_1 = u_2 - x_2 = \frac{20}{3} - \frac{10}{3} = \frac{10}{3}$

Thus $x_1 = x_2 = x_3 = \frac{10}{3}$ and

hence $\text{Max}\{x_1 . x_2 . x_3\} = (10/3)^3$

$= 1,000/27$ Ans

Qn2. Solve by dynamic programming technique:-

Maximize $Z = x_1^2 + x_2^2 + x_3^2$

subject to the constraints $x_1 + x_2 + x_3 \geq 15$

$x_1, x_2, x_3 \geq 0$

Soln. Step-1: To develop the function equation

It is clearly three stage problem which can be defined as follows:—

$s_3 = x_1 + x_2 + x_3 \geq 15$

$s_2 = x_1 + x_2 = s_3 - x_3$

and $s_1 = x_1 = s_2 - x_2$

Let $f_i(s_i)$ be the minimum value of z at the ith stage where $s_i = x_1 + x_2 + \cdots + x_i$, (1,2,3)

where $i = 1, 2, 3$.

Thus the functional equation are

$$f_1(s_1) = \min_{0 \leq x_1 \leq s_1}\{x_1^2\} = \{s_2 - x_2\}^2 \quad \text{— (1)}$$

$$f_2(s_2) = \min_{0 \leq x_2 \leq s_2}(x_1^2 + x_2^2) = \min_{0 \leq x_2 \leq s_2}\{x_2^2 + f_1(s_1)\} \quad \text{— (2)}$$

and $$f_3(s_3) = \min_{0 \leq x_3 \leq s_3}\{x_1^2 + x_2^2 + x_3^2\} = \min_{0 \leq x_3 \leq s_3}\{x_3^2 + f_2(s_2)\} \quad \text{— (3)}$$

Step 2: To solve the functional equations.

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from Eq ① $f_1(s_1) = (s_2 - x_2)^2$

and from Eq ② $f_2(s_2) = \min\limits_{0 \le x_2 \le s_2} \{x_2^2 + f_1(s_1)\}$

$$= \min_{0 \le x_2 \le s_2} \{x_2^2 + (s_2 - x_2)^2\} = \min_{0 \le x_2 \le s_2} \{2x_2^2 - 2s_2 x_2 + s_2^2\}$$

Now $u = 2x_2^2 - 2s_2 x_2 + s_2^2$ is Max or Mini. when

$$\frac{du}{dx_2} = \frac{d}{dx_2}(2x_2^2 - 2s_2 x_2 + s_2^2) = 4x_2 - 2s_2 = 0 \therefore x_2 = \frac{1}{2}s_2$$

$\frac{d^2u}{dx_2^2} = 4$ is positive when $x_2 = \frac{1}{2}s_2 \therefore u$ is minimum

when $x_2 = \frac{1}{2}s_2$

$$\therefore f_2(s_2) = \left(\frac{1}{2}s_2\right)^2 + \left(s_2 - \frac{1}{2}s_2\right)^2 = \frac{1}{2}s_2^2$$

Now $f_3(s_3) = \min\limits_{0 \le x_3 \le s_3} \{x_3^2 + f_2(s_2)\}$

$$= \min_{0 \le x_3 \le s_3} \left\{x_3^2 + \frac{1}{2}(s_3 - x_3)^2\right\}$$

$$\because f_2(s_2) = \frac{1}{2}s_2^2 = \frac{1}{2}(s_3 - x_3)^2.$$

Now let $v = x_3^2 + \frac{1}{2}(s_3 - x_3)^2$ is Max or Mini, when

$$\frac{dv}{dx_3} = 2x_3 - 2 \cdot \frac{1}{2} \cdot (s_3 - x_3) = 3x_3 - s_3 = 0$$

$\therefore \boxed{x_3 = \frac{1}{3}s_3}$

$\frac{d^2v}{dx_3^2} = 3$ which is positive.

$\therefore v$ is minimum when $x_3 = \frac{1}{3}s_3$

$$\therefore f_3(s_3) = \left(\frac{1}{3}s_3\right)^2 + \frac{1}{2}\left(s_3 - \frac{1}{3}s_3\right)^2 = \frac{1}{3}s_3^2.$$

But $s_3 \ge 15$, i.e, minimum $s_3 = 15$.

$\therefore Z$ is minimum when $x_3 = \frac{1}{3}s_3 = 5$.

$x_2 = \frac{1}{2}s_2 = \frac{1}{2}(s_3 - x_3) = 5$, $x_1 = 5$

$\therefore$ Optimal policy is $x_1 = x_2 = x_3 = 5$ & $Z = 75$ Ans

Sem-VIII (Civil) Assignment-1 (3/9) Set-1st

Module-1 Optimization Method:

Q1. What do you mean by optimization? [L1]
Give statement (mathematical formulation) of an optimization problem in terms of decision variables.

Q2. Describe the application of optimization in different Engineering discipline. [L1]

Q3. Give a brief classification of optimization problems. (L1)

Q4. Write about the historical development of optimization. (L1)

Q5. Formulate the following problem into L.P.P (L2)
A resourceful home decorator manufactures two types of Lamps say A and B. Both lamps go through the two technicians; first a cutter and second a finisher. Lamp A requires 2 hours of the cutter's time and 1 hour of the finishers time. Lamp B requires 1 hours of cutters and 2 hours of finishers time. The cutters has 104 hours and finisher 76 hours of available time each month. Profit on one lamp A is Rs 6.00 and on one lamp B is Rs 11.00. Assuming that he can sale all that in the market, he produces, how many of each type of lamps should be manufactured so that the decorator may obtain the best return;

Q6. Find all the basic solutions of the following System (L2)

$$x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + x_2 + 5x_3 = 5$$

and prove that they are non-degenerate.

Sem.-VIII (Civil) Assignment 1 Optimization Method. set-2nd

Module 1.

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Qn 1. How do you use ~~Simplex Method &~~ Revised Simplex method for obtaining the optimal Solution? Apply simplex method to find the inv. of the matrix $\begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$ (L1)

Qn 2 What are optimization methods? Can you recall some of the Ancient Mathematician or Philosopher who gave their great contribution in mathematics. (L1)

Qn 3. What ~~are~~ is theory of optimization & in contrast of constrained and unconstrained optimization) (L1)

Qn 4. What are the three common elements of an optimization problem? (L1)

Q5. Formulate the following problem into LPP. (L2)

A firm can produce three types of cloth A, B and C. Three kinds of wool are required for it, say red wool, green wool and blue wool. One unit length of type A cloth needs 2 yards of red wool and 3 yards of blue wool; one unit length of type B cloth needs 3 yards of red wool, 2 yards of green wool and 2 yards of blue wool, and one unit of type C cloth needs 5 yards of green wool and 4 yards of blue wool. The firm has only a stock of 8 yards of red wool, 10 yards of green wool and 15 yards of blue wool. It is assumed that the income obtained from one unit length of type A cloth is Rs 3.00, of type B cloth is Rs 5.00 and of type C cloth is Rs 4.00. Determine how the firm should use the available materials, so as to maximize the income from the finished cloth.

Q6. Define the following terms :- (L2)

(a) Feasible solution (b) Optimal solution.

(c) A Basic Feasible Solution (B.F.S)

(d) Non-degenerate B.F.S.

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Semester VII (Civil) Assignment-1   **Set-1st**

Module-2. Optimization Method.

Q1. Solve the following LPP:— (L2)

Maximize $Z = 3x_1 + 5x_2 + 4x_3$

Subject to $2x_1 + 3x_2 \leq 8$.

$3x_1 + 2x_2 + 4x_3 \leq 15$

and $x_1, x_2, x_3 \geq 0$.

Q2. Solve the following LPP by using Big-M-Method (L2)

Minimize $Z = 7x_1 + 15x_2 + 20x_3$

Subject to $2x_1 + 4x_2 + 6x_3 \geq 24$

$3x_1 + 9x_2 + 6x_3 \geq 30$

$x_1, x_2, x_3 \geq 0$

Q3. Solve the following LPP by using two-phase method:- (L2)

Min $Z = x_1 + x_2$

Subject to $2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

$x_1, x_2 \geq 0$

~~Q4. Apply simplex method to find the inverse of the matrix~~ $\cancel{\begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}}$

Q4. What do you mean by primal and dual in context of a LPP. Write the dual of the problem ~~(L2)~~ (L1)

Min $Z = 3x_1 + x_2$

s.t. $2x_1 + 3x_2 \geq 2$

$x_1 + x_2 \geq 1$

and $x_1, x_2 \geq 0$.

35. Discuss what is an assignment problem. Write statement (Mathematical formula) of a general assignment problem. Hence solve the following minimal assignment problem by Hungerian Method (L1)

Jobs ↓ / Man →	1	2	3	4
I	12	30	21	15
II	18	33	09	31
III	44	25	24	21
IV	23	30	28	14

36. A company has four plants P_1, P_2, P_3 & P_4, from which it supplies to three markets M_1, M_2, M_3. Determine the optimal transportation plan from the following data giving the plant to market shifting costs, quantities available at each plant and quantities required at each market. (L

Market	Plant P_1	P_2	P_3	P_4	Required at Market
M_1	19	14	23	11	11
M_2	15	16	12	21	13
M_3	30	25	16	39	19
Available at Plant.	06	10	12	15	43.

Semester - VIII (Civil)
Module - 2

Assignment - 1
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Optimization Method

Set - 2nd

Q1. Apply simplex method to find the inverse of the matrix $\begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$ (L2)

Q2. Use the penalty (Big-M) method to solve the following LP problem (L2)

Minimize $Z = 5x_1 + 3x_2$

Subject to the constraints -

$$2x_1 + 4x_2 \le 12$$
$$2x_1 + 2x_2 = 10$$
$$5x_1 + 2x_2 \ge 10.$$
and $x_1, x_2 \ge 0$.

Q3. Prove that dual of the dual of a given primal, is the primal itself. (L1)

Q4. Discuss the Transportation problems to be needed to study. Give its mathematical formulation. (L1)

Q5. Apply the simplex method to solve the following LPP. (L2)

$$\text{Max } Z = 30x_1 + 23x_2 + 29x_3$$
$$\text{s.t. } 6x_1 + 5x_2 + 3x_3 \le 26$$
$$4x_1 + 2x_2 + 5x_3 \le 7$$
and $x_1, x_2, x_3 \ge 0$

Also write the dual of the problem and solve. Read the solution of each problem from the final simplex table of the other.

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Q6. A department has five employees with five jobs to be performed. The time (in hours) each men will make to perform each job is given in the effectiveness matrix.

Jobs	I	II	III	IV	V
A	10	5	13	15	16
B	3	9	18	13	6
C	10	7	2	2	2
D	7	11	9	7	12
E	7	9	10	4	12

How should the jobs be allocated one per employee, so as to minimize the total man hours? (12)

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Sem - VII (Civil) — Assignment - 1 — Set - 3rd

Optimization Method:

Module - 1.

Q1. Give your idea about classical optimization Theory. Do you agree that this method may be suitable for ~~efficient~~ numerical computations? (L1)

Q2. Explain the following ~~linear~~ programming problems and classify them also whether in which category ~~non~~ linear or non-linear do they occur?

1) Integer programming (2) L.P.P. (3) Geometric programming (4) Quadratic Programming (L1)

Q3. Find all the basic solutions to the following problem:-

Maximize $Z = x_1 + 3x_2 + 3x_3$, (L2)

Subject to $x_1 + 2x_2 + 3x_3 = 4$,

$2x_1 + 3x_2 + 5x_3 = 7$.

Also find which of the basic solutions are

(i) basic feasible

(ii) non-degenerate basic feasible, and

(iii) Optimal basic feasible. (L2)

Q4. Differentiate between -

1) Convex sets and Non-Convex sets (L2)

(ii) Solution and Feasible Solution.

(iii) Basic solution and Basic feasible solutions

(iv) optimal basic feasible solution ~~and~~; Degenerate basic feasible solution and Unbounded solutions.

Q5. A manufacturer produces two types of models A and B. Each model of the type A needs 4 hours of grinding and 2 hours of polishing, whereas each model of type A requires 2 hours of grinding and (L2

5 hours of polishing.
The manufacturer has 2 grinders and 3 polishers. Each grinders works for 40 hours a week and each polishers works for 60 hours a week. Profit on A model is Rs 3.00 and on model B is Rs 4.00. Whatever is produced in a week is sold in the market. How should the manufacture allocate his production capacity to the two types of models, so that he may make the maximum profit in a week?

Q6. Give any two application of optimization method in each of the following Engineering discipline
1. Civil 2. Electrical 3. Mechanical.
4. Aeronautics and space-research.
(4)

Sem - VII (Civil) Assignment-1 (329) Set-3rd
Module-2. Optimization Method

Q1. Use the revised simplex method to solve the following LP problem

Maximize $z = 2x_1 + x_2$

Subject to the constraints. $3x_1 + 4x_2 \leq 6$

$6x_1 + x_2 \leq 3$

and $x_1, x_2 \geq 0$ (L2)

write its dual.

Q2. Use the simplex method to solve the following LP problem

Maximize $z = 3x_1 + 5x_2 + 4x_3$

Subject to the constraints

$2x_1 + 3x_2 \leq 8$

$2x_2 + 5x_3 \leq 10$

$3x_1 + 2x_2 + 4x_3 \leq 15$ (L2)

and $x_1, x_2, x_3 \geq 0$

Q3. Show that the feasible solution:

$x_1 = 1, x_2 = 0, x_3 = 1$; $z = 6$ to the system of equations $x_1 + x_2 + x_3 = 2$ (L2)

$x_1 - x_2 + x_3 = 2$

with Maximum $z = 2x_1 + 3x_2 + 4x_3$ is not basic.

Q4. What is difference between simplex method and revised simplex method? When and where the two should be applied?

Use the revised simplex method to solve the following LP problem.

Maximize $z = 6x_1 - 2x_2 + 3x_3$

subject to the constraints $2x_1 + x_2 + 2x_3 \leq 2$

$2x_1 + 4x_3 \leq 4$

and $x_1, x_2, x_3 \geq 0$

(L4)

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85 Use the dual simplex method to solve the LP problem

Maximize $Z = -2x_1 - x_3$

subject to the constraints

$x_1 + x_2 - x_3 \geq 5$

$x_1 - 2x_2 + 4x_3 \geq 8$

and $x_1, x_2, x_3 \geq 0$. (L2)

86. A company has three production facilities S_1, S_2 and S_3 with production capacity of 7, 9 and 18 units (in 100s) per week of a product respectively. These units are to be shipped to four warehouses D_1, D_2, D_3 & D_4 with requirement of 5, 6, 7 and 14 units (in 100's) per week, respectively. The transportation cost (in rupees) per unit between factories to warehouses are given in the table below: (L2)

Facilities	D_1	D_2	D_3	D_4	Capacity
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18.
Demand					

Formulate this transportation problem as an LP model to minimize the total transportation cost. by any of the following method:—

I) North-West Corner Method (NWCM)

II) Least Cost Method and

III) Vogel's Approximation (or Penalty) (VAM) Method L

Objective Question (331) Test-1

Open elective - II , Sub - Optimization Methods
Sub. Code - 01HS 2185

Write true or False :-

Q1. The optimal solution of the dual problem is readily available from the optimal solution to a primal problem (True/False)

Q2. For a 4- variable and 5-constraint primal problem, the dual would be a 5-variable and 4-Constraint problem (True/False)

Q3. An unbalanced transportation problem must be converted into a balanced problem before solving it (True/False)

Q4. A transportation problem is a special type of linear programming problem (True/False)

Ans. Q(1) – True

Q(2) – True

Q(3) – True

Q(4) – True.

Open elective-II (OM) (Subjective Qns) Internal Test-I

Qs. 6. L2 [Understanding] Sub Code - 01HS 2851

Maximize (total profit) $Z = 4x_1 + 3x_2$

Solution on Page 140 J.K. Sharma

1. Availability of time constraint.

$$2x_1 + x_2 \le 1000.$$

2. Supply of leather constraint Module-2

$$x_1 + x_2 \le 800$$

3. Buckles availability constraints

$$x_1 \le 400, \quad x_2 \le 700, \quad x_1, x_2 \ge 0$$

Q7. L2 [Understanding]

A manufacturer of articles produce chairs and tables. The manufacturer knows that a table requires 3 unit of wood and 1 unit of labour and similarly a chair requires 2 unit of wood and 2 unit of labour. The profit on each table and chair is Rs 20 and Rs 16 respectively. The total available resources are 150 units of wood and 75 units of labor. Formulate this an LPP to maximize profits.

A.P. Verma P-25 Example-2.1

(Module-1)

Q8. L3 [Application]

A firm produces 3 product these product are processed on three (3) different machines. The time required to manufacture one unit of each of the 3 products and the daily capacity of the 3 machines is given below. Machine Time per unit (in minutes) Machine capacity (minutes/day)

Example 2.3 Page-26 A.P. Verma

Machines	product-1	product 2	product 3	Machine capacity
M_1	2	3	2	440
M_2	4	—	3	470
M_3	2	5	—	430

It is required to determine the daily no. of units to be manufactured for each products. The profit per unit for product 1, 2, 3 is Rs 4, Rs 3 and Rs 6. It is assumed that all the product are consumed in the market. Formulate the LP model.

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Module-1

Q. 9. L3 [Application]

A computer centre has three expert programmers. The centre wants three application programmes to be developed. The head of the computer centre, after studying carefully the programmes be developed estimates the computer time in minutes required by the experts for the application programmes as follows:—

Chap 13 from exercise unsolved Q11 Page-579 R.K-Gupta.

		Programmers		
		A	B	C
Programmes	1	120	100	80
	2	80	90	110
	3	110	140	120

Assign the programmes to the programmers in such a way that the Total computer time is minimum.

Ans: 1. Programmes → Progs.: 1 → C, 2 → B, 3 → A. Min. Time = 280M

(Module-2)

Q10. (L4/L3) [Analytical/Numerical]

Solve the LP problem using simplex method:—

Maximize $Z = 3x_1 + 5x_2 + 4x_3$

Subject to the constraints $2x_1 + 3x_2 \le 8$

$2x_2 + 5x_3 \le 10$

$3x_1 + 2x_2 + 4x_3 \le 15$

and $x_1, x_2, x_3 \ge 0$

Soln

Example-3

Page-314

OR by R.K-Gupta.

Also solved on 136 J.K. Sharma Ex-4.1

(Module-3)

Sem-8th

Solution of Subjective Questions of Internal Test-I (OM)

Q 6 L2 [Understanding]

Maximize (total profit) $Z = 4x_1 + 3x_2$

1. Availability of time constraint

$$2x_1 + x_2 \le 1000$$

2. Supply of Leather constraint

$$x_1 + x_2 \le 800$$

3. Buckles availability constraints

$$x_1 \le 400, \; x_2 \le 700, \; x_1, x_2 \geq 0$$

Soln:. Standard form: Introducing slack variables x_1, x_2, x_3 and x_4 to convert inequality constraints to equality. The LP model becomes:

Maximize $Z = 4x_1 + 3x_2 + 0x_1 + 0s_2 + 0s_3 + 0s_4$

Subject to the constraints

$$2x_1 + x_2 + x_1 = 1000$$
$$x_1 + x_2 + x_3 = 800$$
$$x_1 + x_3 = 400$$
$$x_2 + x_4 = 700$$

and $x_1, x_2, x_3\ x_4 \geq 0$

Solution by Simplex Method:– An initial feasible soln is obtained by setting $x_1 = x_2 = 0$. Thus, the initial solution is $s_1 = 1000$, $s_2 = 800$, $s_3 = 400$, $s_4 = 700$ and Max $Z = 0$. Thus the solution can also be read from the initial simplex Table ~~(I)~~ as follows:–

Table-01 Initial Solution

Profit per Unit C_B	Variables in Basis B	Solution Values $b(=X_B)$	$C_j \rightarrow$ 4 x_1	3 x_2	0 s_1	0 s_2	0 s_3	0 s_4	Min Ratio X_B/x_1
0	s_1	1,000	2	1	1	0	0	0	1,000/2 = 500
0	s_2	800	1	1	0	1	0	0	800/1 = 800
0	s_3	400	1	0	0	0	1	0	400/1 = 400 →
0	s_4	700	0	1	0	0	0	1	not defined
Z = 0		z_j	0	0	0	0	0	0	
		$c_j - z_j$	4 ↑	3	0	0	0	0	

In table-01, since $c_1 - z_1 = 4$ is the largest positive number, we apply the following row operations in the same manner as discussed earlier to get an improved basic feasible solution by entering variable x_1 into the basis and removing variable S_3 from the basis

$R_3(\text{new}) \rightarrow R_3(\text{old}) \div 1$ (key element)

$R_2(\text{new}) \rightarrow R_2(\text{old}) - R_3(\text{new})$

$R_1(\text{new}) \rightarrow R_1(\text{old}) - 2R_3(\text{new})$

The new solution is shown in Table 02

Table-02 An improved solution

Profit per Unit C_B	Variables in Basis B	Soln Values $b(=X_B)$	$C_j \rightarrow$ 4 x_1	3 x_2	0 s_1	0 s_2	0 s_3	0 s_4	Min Ratio X_B/x_2
0	s_1	200	0	1	1	0	-2	0	200/1 = 200 →
0	s_2	400	0	1	0	1	-1	0	400/1 = 400
4	x_1	400	1	0	0	0	1	0	—
0	x_4	700	0	1	0	0	0	1	700/1 = 700
Z = 1,600		z_j	4	0	0	0	4	0	
		$c_j - z_j$	0	3 ↑	0	0	-4	0	

The solution shown in Table-02 is not optimal because $c_2 - z_2 > 0$ in x_2-column. Thus, again applying the following row operations to get a new solution by entering variable x_2 into the basis and removing variable s_1 from the basis,

$R_1(new) \rightarrow R_1(old) \div 1$ (Key element)

$R_4(new) \rightarrow R_4(old) - R_1(new)$!

The improved solution is shown in Table-03 below.

Table 03 Improved Solution

Profit per unit c_B	Variables in Basis B	$c_j \rightarrow$ Solution Value $b(=x_B)$	4 x_1	3 x_2	0 s_1	0 s_2	0 s_3	0 s_4	Min Ratio x_B/s_3
3	x_2	200	0	1	1	0	-2	0	—
0	s_2	200	0	0	-1	1	1	0	$200/1 = 200 \rightarrow$
4	x_1	400	1	0	0	0	1	0	$400/1 = 400$
0	s_4	500	0	0	-1	0	2	1	$500/2 = 250$
$Z = 2{,}200$		z_j	4	1	3	0	-2	0	
		$c_j - z_j$	0	0	-3	0	2	0	

The solution shown in Table-03 is not optimal because $c_5 - z_5 > 0$ in s_3-column. Thus, again applying the following row operation to get a new solution by entering variable s_3 into the basis and removing variable s_2 from the basis:

$R_2(new) \rightarrow R_2(old) \div 1$ (Key element)

$R_3(new) \rightarrow R_3(old) - R_2(new)$;

$R_1(new) \rightarrow R_1(old) + 2R_2(new)$ and $R_4(new) \rightarrow R_4(old) - 2R_2(new)$

The new improved soln is shown in the table-04 below:—

Table-04 (34) Optimum solution

Profit per Unit C_B	Variable in Basis B	Soln Values $b(=x_B)$	x_1 (4)	x_2 (3)	s_1 (0)	s_2 (0)	s_3 (0)	s_4 (0)
3	x_2	600	0	1	-1	2	0	0
0	s_3	200	0	0	-1	1	1	0
4	x_1	200	1	0	1	-1	0	0
0	s_4	100	0	0	1	-2	0	1
$Z = 2,600$		Z_j	4	3	1	2	0	0
		$C_j - Z_j$	0	0	-1	-2	0	0

Since all $C_j - Z_j \le 0$ corresponding to non-basic variables columns, the current solution cannot be improved further. This means that the current basic solution which is feasible is also the optimal soln.
Thus, the company must manufacture $x_1 = 200$ belts of type A and $x_2 = 600$ belts of type B to obtain maximum profit of Rs 2,600/- Ans

Q7. L2 [Understanding]
A manufacturer of articles produce chairs and table. The manufacturer knows that a table requires 3 unit of wood and 1 units of Labour and similarly a chair requires 2 unit of wood and 2 units of Labour. The profit from each table and chair is Rs 20 and Rs 16 respectively. The total available resources are 150 units of wood and 75 units of labour. Formulate ~~the~~ an LPP to maximize profits.

Soln:- Let x_1 represent table and x_2 chair
Wood required for x_1 tables $= 3x_1$ units
Wood required for x_2 chairs $= 2x_2$ units
Labour required for x_1 tables $= 1x_1$ units
Labour required for x_2 chairs $= 2x_2$ units.

Total wood required (342) $= 3x_1 + 2x_2$ which cannot exceed the available resource of ~~labour~~ wood 150 of units.
Total labour required $= x_1 + 2x_2$ which cannot exceed the available resource of labour 75 of units. Total profit from x_1 tables and x_2 chairs $= 20x_1 + 16x_2$
Also the numbers of tables and chairs cannot be negative so $x_1 \geq 0$ and $x_2 \geq 0$.
Thus the maker of furniture has the following problem at hand

Maximize $Z = 20x_1 + 16x_2$ (obj. fn)

~~Subject to~~
$$\left.\begin{aligned} 3x_1 + 2x_2 &\leq 150 \\ x_1 + 2x_2 &\leq 75 \\ x_1 \geq 0,\ x_2 &\geq 0 \end{aligned}\right\}$$
Constraints and $x_1 \geq 0$, $x_2 \geq 0$ are non-negativity constraints

— Ans —

Q8. L3 [Application]

~~A firm can manufacture three types of cloth namely A, B and C. Three types of wool are required.~~

~~A firm manufacture three type of products A, B and C. The profits from A, B and C are Rs 6, Rs 4 and Rs 8 respectively. The firm has two machines and given below is the required processing time (in minutes) for each machine on each product.~~

Machine	Product A	Product B	Product C
X	8	6	10
Y	4	4	8

Q8 L3 [Application] (343)

A firm produces 3 product. These product are processed on three(3) machines. The time required to manufacture one unit of each of the 3 products and the daily capacity of the 3 machine is given below:
Machines Time per unit (in minutes) Machine capacity (minutes)

Machines	product 1	product 2	product 3	Machine Capacity
M_1	2	3	2	440
M_2	4	–	3	470
M_3	2	5	–	430

It is required to determine the daily no. of units to be manufactured for each products. The profit per unit ~~of~~ for the products 1, 2, 3 is Rs 4, Rs 3 and Rs 6 respectively. It is assumed that all the products are consumed in the market. Formulate the LP. model that will maximize the daily profit.

Sol. Formulation of Linear programming Model :—

Step-1 From the study of the situation we find the key-decision to be made. In this connection working for variables helps considerably. In the given situation key decision is to decide the no. of units of products 1, 2 and 3 to be produced daily.

Step-2 Assuming symbols ~~1, 2 and 3~~ for variable quantities noticed in step-1. Let the no. of units of product 1, 2 and 3 manufactured be x_1, x_2 and x_3.

Step-3 We express the feasible alternatives mathematically in terms of variables. Feasible alternatives are those which are physically, economically and financially possible. In the given situation feasible alternatives are sets of values of x_1, x_2 & x_3. Where $x_1, x_2, x_3 \geq 0$.
Since negative production has no meaning and is not feasible.

Step-4 We mention the objective quantitatively and express it as a linear function of variables. In the present situation, objective is to maximize the profit.

i.e, Maximize $Z = 4x_1 + 3x_2 + 6x_3$

Step-5. We put into words the influencing factors or constraints. These occurs generally because of constraints on availability (of resources) or requirements (or demands).
So we express these constraints also as linear eqns inequalities in terms of variables.
So here the constraints are on the machine capacities and can be mathematically expressed as

$$2x_1 + 3x_2 + 2x_3 \leq 400$$
$$4x_1 + 0x_2 + 3x_3 \leq 470$$
$$2x_1 + 5x_2 + 0x_3 \leq 430$$

Therefore the complete mathematical LP model for the problem can be written as.

Maximize $Z = 4x_1 + 3x_2 + 6x_3$ subject to the constraints:

$$2x_1 + 3x_2 + 2x_3 \leq 440$$
$$4x_1 + 3x_3 \leq 470$$
$$2x_1 + 5x_2 \leq 430$$

and $x_1, x_2, x_3 \geq 0$.

Q9 L.3 [Application]. 345

A computer centre has three expert programmers. The centre wants three application programmes to be developed. The head of the computer centre, after studying carefully the programmes be developed estimates the computer time in minutes required by the experts for the application programmes as follows:-

		Programmes		
		A	B	C
programmers	1	120	100	80
	2	80	90	110
	3	110	140	120.

Assign the programmes to the programmers in such a way that the total computer time is minimum.

Ans: Programmers → Programmes

1 → C

2 → B

3 → A

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Q10. (L4/L3) [Analytical / Numerical]

Solve the LP problem using simplex method:

Maximize $Z = 3x_1 + 5x_2 + 4x_3$

Subject to the constraints
$$2x_1 + 3x_2 \leq 8$$
$$2x_2 + 5x_3 \leq 10$$
$$3x_1 + 2x_2 + 4x_3 \leq 15$$
and $x_1, x_2, x_3 \geq 0$

Sol: Solve in the handout-notes already.

B.Tech Civil 6th sem. Internal Test-II 349

Sub. Optimization method, Sub. code-011tS2857

(Objective Question for 2nd Test)

Module-1

Q1. Formal optimization approach on LP was first introduced by Leonid Katorovich in 1939

Q2. In Geometric programming problem both the objective function and Constraints are expressed as polynomial vector X

Q3. Quadratic programming is a NLP with a quadratic objective function and linear constraints.

Q4. Direct search is the continuous but non-differental optimization method.

Module-2

Mark the following statement as T(True/False)

Q1. For a linear programming problem to be unbounded, its feasible region must be unbounded

Q2. Successive solution obtained in the simplex alogorithm always yield higher and higher values of z'

Q3. The dual to the dual of an LP is the primal LPP itself

Q4. The initial solution obtained in the Transportation problem by the least-cost method would invariably be optimal.

~~Module~~

CamScanner

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Ans:. Module-1

Q1 - True(T)

Q2 - False(F)

Q3 - True(T)

Q4 - True(T)

Ans. Module-2

Q1 - True (T)

Q2 - False (F)

Q3 - True(T)

Q4 - False (F)

———x ———x ———x———

(351)

Subjective type. Internal Test-II Branch Civil (8th Sem)

Sub:- Optimization Methods. Sub Code - 011-3-2851

Level-II [Understanding]

Qn 1 Discuss the problem of optimizing a continuous and differentiable multivariable function subject to equality constraints.

Qn 2. What are the methods commonly used to solve the continuous and differentiable multivariable function subject to equality constraints?

Q3. Define the following terminologies in context of dynamic programming problems:-
(I) Stages (II) states (III) Return function

Q4. What is Dynamic programming? Differentiate between a Linear programming problem and Dynamic programming problem. Write Bellman's principle of optimality in Dynamic programming.

Level-III [Application]

Q1. Obtain the necessary condition for the optimality of the following problems:-

$$\text{Minimize } f(x_1, x_2) = 3e^{2x_1+1} + 2e^{x_2+5}$$

Q2. Use Lagrange Multiplier Method to find necessary and sufficient condition for a problem with $n=3$ and $m=1$, when n denotes the no. of decision variables and m are the no. of equality constraints.

A positive quantity c is to be divided into n parts in such a way that the product of the n parts is to be maximum. Obtain the the optimal subdivision.

Write any three characteristics of Dynamic programming problems. Give steps of its method of solution.

Level III/IV [Analytical/Numerical]

Q1. Consider the function $f(x) = x_1 + 2x_2 + x_1x_2 - x_1^2 - x_2^2$. Determine the maximum or minimum point (if any) of the function.

Q2. Obtain the necessary condition for the optimum solution of the following problems:

$$\text{Minimize } f(x_1, x_2) = e^{2x_1+1} + e^{x_2+5}$$

Q3. Determine x_1, x_2, x_3 so as to Maximize $Z = x_1 x_2 x_3$ subject to the constraints $x_1 + x_2 + x_3 = 10$ and $x_1, x_2, x_3 \geq 0$.

Q4. By dynamic programming technique, solve the problem

Maximize $Z = x_1^2 + x_2^2 + x_3^2$

subject to the constraints $x_1 + x_2 + x_3 \geq 15$

$x_1, x_2, x_3 \geq 0$.

353 LEVEL-II (Understanding)

Q1. Discuss the problem of optimizing a continuous and differentiable multivariable function subject to equality constraints.

Soln. If the objective function of a NLPP is continuous and differentiable and all the constraints are equality constraints (i.e, equations), then they can be solved by the use of Lagrangian multipliers.

There are three cases :-.

(i) Two decision variables and one equality constraints

(ii) n decision variables and one equality constraints and

(iii) n decision variable and two equality constraints.

and these are all already discussed in hand out notes. including Qn ② which is

Q2. Q2.. What are the methods commonly used to solve the continuous and differentiable multivariable function subject to equality constraints?

Q3. define (i) stages (ii) states (iii) Return fn.

Ans All discussed in the chapter of Dynamic progrg

Q4. What are dynamic programming problem? Differentiate between a LPP & DPP. Write Bellman's principle of optimization in dynamic programming?

Ans These are already been discussed in the Dynamic Programming chapter at length.

Rest all questions ~~for~~ (354) a part of the hand out notes of concerned chapters.

Level - III (Application)

Q1 Soln. Solved on page - 124

Q2